N1 $\mathbb{P}\{\xi = 1\} = 0.9$

$\mathbb{P}\{\xi = 2\} = 0.1 \cdot 0.9$

$\mathbb{P}\{\xi = 3\} = 0.01$ – ?

$\xi$:

| 1 | 2 | 3 |
|---|---|---|
| 0.9 | 0,09 | 0,01 |

N2 $\xi \sim Pois\ \lambda_1,\ \eta \sim Pois\ \lambda_2$

$\zeta = \xi + \eta \qquad \mathbb{P}\{\xi = k\} = \frac{e^{-\lambda}\lambda^k}{k!}$

$$\mathbb{P}\{\zeta = n\} = \mathbb{P}\{\xi + \eta = n\} = \sum_{k=0}^{n} \mathbb{P}\{\xi = k\}\mathbb{P}\{\eta = n-k\} =$$

$$= \sum_{k=0}^{n} \frac{e^{-\lambda_1}\lambda_1^k}{k!} \cdot \frac{e^{-\lambda_2}\lambda_2^{n-k}}{(n-k)!} = e^{-(\lambda_1+\lambda_2)} \sum_{k=0}^{n} \frac{n!\ \lambda_1^n \lambda_2^{n-k}}{k!(n-k)!\,n!} =$$

$$= e^{-(\lambda+\lambda_2)} \cdot \frac{1}{n!} \sum_{k=0}^{n} C_n^k \lambda_1^n \lambda_2^{n-k} =$$

$$= \frac{(\lambda_1+\lambda_2)^n e^{-(\lambda_1+\lambda_2)}}{n!}$$ – снова Пуассон

$$\mathrm{cov}(\xi, \xi+n) = E\xi(\xi+\eta) - E\xi E(\xi+n) =$$

$= E\xi^2 + E\xi\eta - E\xi E\xi - E\xi E\eta$

$= \operatorname{cov}(\xi, \eta) + D\xi = 0 + \lambda_1 = \lambda_1$

N3

$$p_\xi(x) = \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-1)^2}{4}}$$

$$\mathbb{P}\{-0.5 \leq \xi \leq 3\} = F_\xi(3) - F(-0,5)$$

$$= \Phi_0\left(\frac{3-1}{\sqrt{2}}\right) - \Phi_0\left(\frac{\overset{?}{-}0,5-1}{\sqrt{2}}\right) \approx 0,4207 - 0,3554$$

$$= 0,0653$$

$$\eta = e^\xi$$

$$F_\eta(t) = \mathbb{P}\{e^\xi < t\} = \mathbb{P}\{\xi < \ln t\} =$$

$$= F_\xi(\ln t)$$

$$p_\eta(t) = F'_\eta(t) = \frac{1}{t} p_\xi(t) = \frac{1}{2\sqrt{\pi}\, t} e^{-\frac{(t-1)^2}{4}}$$

$$E\eta = Ee^\xi = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} e^x e^{-\frac{(x-1)^2}{4}} dx =$$

$$= \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{\frac{-x^2+6x-1}{4}} dx = \frac{e^2}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\frac{(x-3)^2}{4}} dx =$$

$$= e^2 \int\limits_{-\infty}^{+\infty} \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-3)^2}{4}} dx = e^2$$

$$E\eta^2 = \frac{1}{2\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} e^{2x} e^{-\frac{(x-1)^2}{4}} dx =$$

$$= \frac{e^6}{2\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} e^{-\frac{(x-5)^2}{4}} = e^6$$

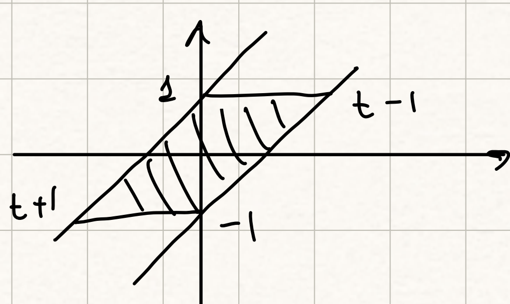
$$D\eta = e^6 - e^4$$

№4. $\xi \sim U[-1,1]$, $\eta \sim U[-1,1]$

$$\zeta = \xi - \eta$$

$$F_\zeta(t) = \int\limits_{-\infty}^{+\infty} p_\xi(u)\, p_\eta(u-t)\, du = I$$

$$\begin{cases} -1 \leq u \leq 1 \\ -1 \leq u - t \leq 1 \end{cases} \Rightarrow \begin{cases} -1 \leq u \leq 1 \\ t-1 \leq u \leq t+1 \end{cases}$$



$$I = \begin{cases} \int\limits_{-1}^{t+1} \frac{1}{4}\,du, & t \in [-2,0] \\ \int\limits_{t-1}^{1} \frac{1}{4}\,du, & t \in [0,2] \end{cases} =$$

$$= \begin{cases} \frac{t}{4} + \frac{1}{2}, & t \in [-2,0] \\ -\frac{t}{4} + \frac{1}{2}, & t \in [0,2] \end{cases}$$

$$p_\xi(x) = \begin{cases} \frac{x}{4} + \frac{1}{2}, & x \in [-2,0] \\ -\frac{x}{4} + \frac{1}{2}, & x \in [0,2] \\ 0, & x \notin [-2,2] \end{cases}$$

$$E\xi = \int\limits_{-\infty}^{+\infty} x p_\xi(x)\,dx = \int\limits_{-2}^{0} \left(\frac{x^2}{4} + \frac{x}{2}\right) dx + \int\limits_{0}^{2} \left(-\frac{x^2}{4} + \frac{x}{2}\right) dx =$$

$$= \int\limits_{-2}^{0} \frac{x^2}{4}\,dx + \int\limits_{-2}^{2} \frac{x}{2}\,dx - \int\limits_{0}^{2} \frac{x^2}{4}\,dx =$$

$$= \frac{x^3}{12}\Big|_{-2}^{0} + \frac{x^2}{4}\Big|_{-2}^{\overset{=0}{2}} - \frac{x^3}{12}\Big|_{0}^{2} = \frac{8}{12} - \frac{8}{12} = 0$$

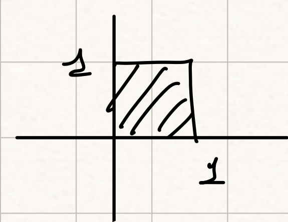
$$E\xi^2 = \int\limits_{-2}^{0} \frac{x^3}{4}\,dx + \int\limits_{-2}^{2} \frac{x^2}{2}\,dx - \int\limits_{0}^{2} \frac{x^3}{4}\,dx =$$

$$= \frac{x^4}{16}\Big|_{-2}^{0} + \frac{x^3}{6}\Big|_{-2}^{2} - \frac{x^4}{16}\Big|_{0}^{2} =$$

$$= -1 + \frac{8}{6} + \frac{8}{6} - 1 = \frac{16}{6} - \frac{12}{6} = \frac{2}{3}$$

$$D\xi = \frac{2}{3}$$

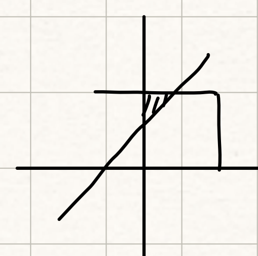
N5 $\quad p(x,y) \quad \xi, y$




$$p(x,y) = \begin{cases} cxy, & 0 \leq x, y \leq 1 \\ 0, & \text{иначе} \end{cases}$$

$$\iint\limits_{0 \leq x, y \leq 1} cxy\,dxdy = \int_0^1 dx \int_0^1 cxy\,dy = c\int_0^1 x\,dx \int_0^1 y\,dy =$$

$$= \frac{c}{2}\int_0^1 x\,dx = \frac{c}{4} = 1 \Rightarrow c = 4$$



$$\mathbb{P}\{\eta > \xi + \tfrac{1}{2}\} = \iint\limits_{y > x + \frac{1}{2}} 4xy\,dxdy =$$

$$= \int_0^{1/2} dx \int_{x+\frac{1}{2}}^{1} 4xy\,dy = \int_0^{1/2} x \int_{x+\frac{1}{2}}^{1} 4y\,dy =$$

$$= -2\int_0^{1/2} x^3 dx - 2\int_0^{1/2} x^2 dx + \frac{3}{2}\int_0^{1/2} x dx =$$

$$= -\frac{x^4}{2}\Big|_0^{1/2} - \frac{2x^3}{3}\Big|_0^{1/2} + \frac{3x^2}{4}\Big|_0^{1/2} =$$

$$= -\frac{1}{32} - \frac{1}{12} + \frac{3}{16} = \frac{7}{96}$$